BEAM THEORY – AXIAL DISPLACEMENTS

STRUCTURAL MECHANICS

The ERAMCA Project

Environmental Risk Assessment and Mitigation on Cultural Heritage assets in Central Asia

v2022317

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Lecturer/students objectives

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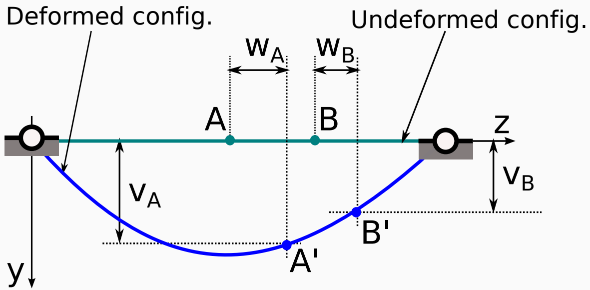
- Present the deformation of plane elastic beams in terms of longitudinal displacements of the longitudinal axis.
- Understand the mathematical model for the elastic curve and the solution method taking into account the proper boundary conditions. Apply the theory to calculate displacements and rotations to beams under different loading and boundary conditions.

Introduction



The axial (longitudinal) displacements (z direction) $w(z)$ and transverse (y direction) $v(z)$ can be calculated form kinematics and constitutive equations

Longitudinal displacements (2/2)

It is possible to obtain uncoupled equations:

- axial displacements $w(z)$:
  1. relationship between the first derivative of $w(z)$ and $\frac{N(z)}{EA}$
  2. relationship between the second derivative of $w(z)$ and $\frac{p(z)}{EA}$
- transverse displacements $v(z)$:
  1. relationship between the second derivative of $v(z)$ and $\frac{M(z)}{EI_x}$
  2. relationship between the fourth derivative of $v(z)$ and $\frac{q(z)}{EI_x}$

Here, the calculation of the axial displacements $w(z)$ is examined; the calculation of the transverse displacements $v(z)$ is studied in the ‘Deflection” lecture.

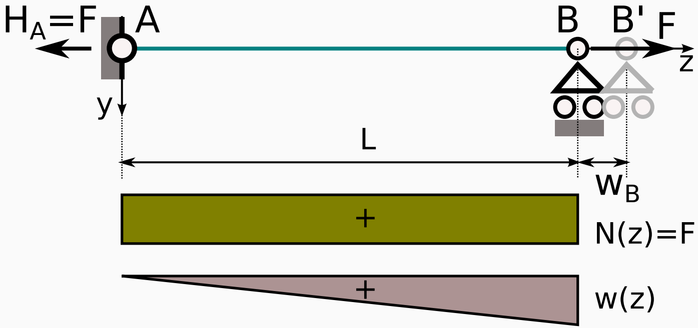
The relationship between axial displacement $w(z)$ and normal force $N(z)$ is given by:

$$\varepsilon_0(z) = \frac{\mathrm{d}w(z)}{\mathrm{d}z} = \frac{N(z)}{K_{A,e}} = \frac{N(z)}{EA}$$

If the normal force $N(z)$ and E e A are constant along z, it is easy to integrate the previous equation to calculate the elongation $w(z)$:

$$w(z) = \frac{N}{EA} z + C_1$$

and to associate one boundary condition.



The normal force is $N(z) = +F$; the boundary condition:

$$w(z = 0) = w_A = 0 \implies C_1 = 0$$

It is obtained:

$$w(z) = \frac{F}{EA} z$$

The displacement of point B is:

$$w_B = w(z = L) = +\frac{F L}{E A}$$

Axial stiffness

Term $K_{a,rod} = \frac{EA}{L}$ represents the stiffness of the element of length L subjected to an axial force ($F = K_{a,rod}\, w_B$)

Axial stiffness of rods

Rods with different area and length loaded in tension:

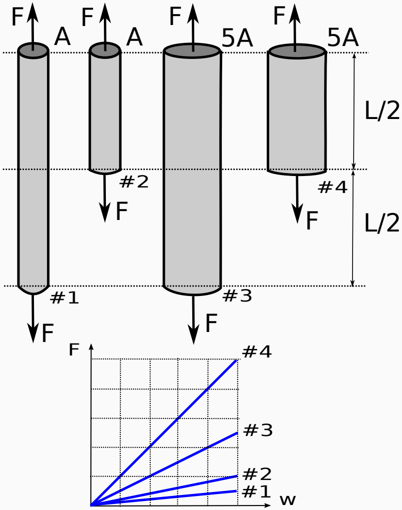
#1: $K_1 = \frac{EA}{L}$

#2: $K_2 = \frac{EA}{L/2} = 2\frac{EA}{L}$

#3: $K_3 = \frac{E(5A)}{L} = 5\frac{EA}{L}$

#4: $K_4 = \frac{E(5A)}{L/2} = 10\frac{EA}{L}$

The stiffer element is #4

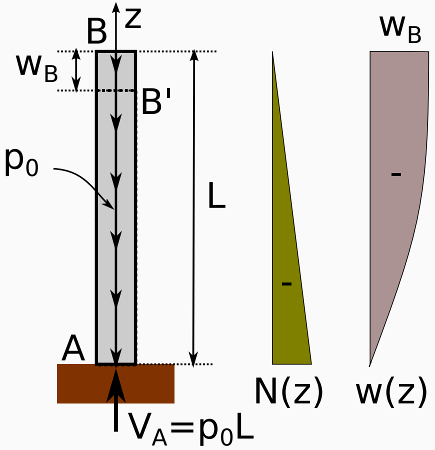


The normal force due to self-weight $p(z) = -\rho\, g\, A = -p_0$ is $N(z) = -p_0\,(L - z)$, so that (where E and A constant along z):

$$\frac{\mathrm{d}w(z)}{\mathrm{d}z} = \frac{N(z)}{E\,A} = \frac{-p_0\,(L - z)}{E\,A}$$

ρ is the material density and g ($9.81\,\mathrm{m/s^2}$) the acceleration due to gravity. It is obtained:

$$w(z) = -\frac{p_0}{E\,A}\left(Lz - \frac{z^2}{2}\right) + C_1$$



The boundary condition is:

$$w(z=0) = w_A = 0 \implies C_1 = 0$$

hence:

$$w(z) = -\frac{p_0}{EA}\left(Lz - \frac{z^2}{2}\right)$$

The displacement of B, w_B, is now:

$$w_B = w(z=L) = -\frac{p_0 L^2}{2EA} = -\frac{\rho g L^2}{2E}$$