BEAM THEORY – MATERIAL PROPERTIES

STRUCTURAL MECHANICS

The ERAMCA Project

Environmental Risk Assessment and Mitigation on Cultural Heritage assets in Central Asia

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Table of Contents

LECTURER/STUDENTS OBJECTIVES

Lecturer/students objectives

- Present the physical meaning of the material properties.
- Understand the relationships between stresses and strains.

Introduction

Aim of the lesson

The aim of the lecture is to provide the information necessary to understand:

- relations between stress and strains, i.e., stress-strain constitutive equations for a given point of a beam.

PHYSICAL MEANING OF MATERIAL PROPERTIES

Remarkable points:

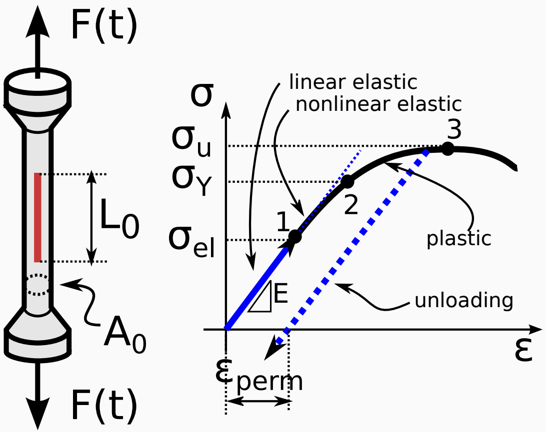
1: Proportionality limit σ_{el}

2: Elastic limit or yield strength σ_Y

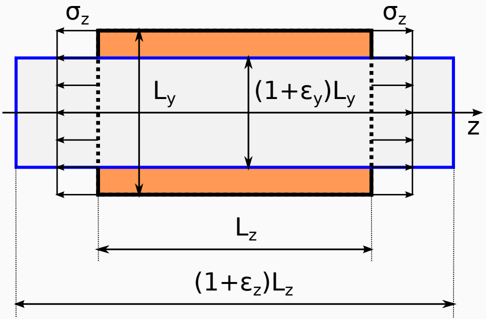
3: Ultimate strength σ_u

where $\sigma = {}^{F}/_{A_0}$ (see the lecture about normal stress) and $\varepsilon = {}^{\Delta L}/_{L_0}$

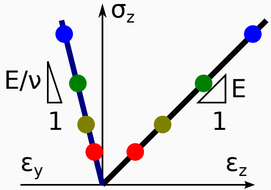
Yield point: limit of elastic behavior and the beginning of plastic behavior



- Elastic modulus or Young's modulus E: slope of the tensile test diagram (vertical axis: σ_z; horizontal axis: elongation $\varepsilon_z = {}^{\Delta L_z}/_{L_z}$ in the direction of σ_z):



$$E = \frac{\sigma_z}{\varepsilon_z}$$



Poisson's ratio and tangential modulus

- Poisson's ratio ν: ratio between transversal and longitudinal strains:

$$\nu = -\frac{\varepsilon_y}{\varepsilon_z} = -\frac{\Delta L_y / L_y}{\Delta L_z / L_z}$$

- Shear (or tangential) modulus G:

$$G = \frac{E}{2\,(1+\nu)}$$

This is valid for isotropic materials

Isotropic and homogeneous materials

- A materials is considered isotropic if their properties were independent of direction (they can be different for different points)

  Steel is an isotropic material, wood is anisotropic, i.e., its properties depend upon direction (fiber growth direction)

- A material is homogeneous if their properties does not change upon the points

If the material is isotropic the elastic properties...

... at each point are only two (E and ν or E or G)

If the material is homogeneous...

...the elastic properties are the same for all points

The elastic parameters E, ν and G are bounded, they must respect the following inequalities:

$$E > 0; \quad G > 0; \quad -1 < \nu < \frac{1}{2}$$

Negative Poisson ratio?

The Poisson's ratio ν can assume negative values, for materials like foams

Some magnitude order

Table from L. Gambarotta, L. Nunziante, A. Tralli, *Scienza delle Costruzioni*, McGraw Hill, 2003

	Elasticity modulus E GPa	Shear modulus G GPa	Poission ratio ν –
Steel	207–210	82	0.26–0.33
Aluminum	69–70	25–26	0.26–0.33
Concrete	20–35	12	0.15–0.16
Iron	180–210	78–81	0.30
Wood[1]	8–15		
Marble	40–70	26	0.15
Brass	100–120	37	0.36
Copper	105–124	44	0.35–0.36
Glass	70	29	0.22

[1]Measured along fiber direction

Material	Yield strength MPa	Density g/cm^3
ASTM A36 steel	250	7.87
Steel (prestressing strands)	1650	7.85
Titanium alloy (6% Al, 4% V)	830	4.51
Aramid (Kevlar or Twaron)	3620	1.44

LINEAR ELASTIC ISOTROPIC STRESS-STRAIN RELATIONSHIP

Proportion between stress and strain:

- direct (stress as function of strain):

$$\sigma_z = E\,\varepsilon_z$$

$$\tau_{zx} = G\,\gamma_{zx} \qquad \tau_{zy} = G\,\gamma_{zy}$$

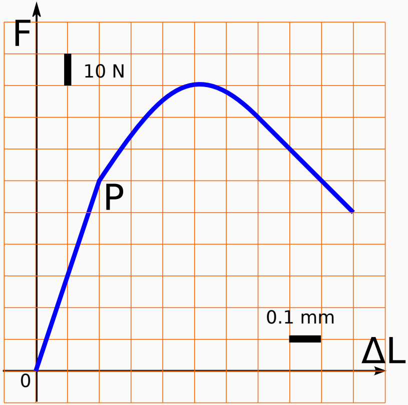
- inverse (strain as function of stress):

$$\varepsilon_x = -\frac{\nu}{E}\sigma_z \qquad \varepsilon_y = -\frac{\nu}{E}\sigma_z \qquad \varepsilon_z = \frac{\sigma_z}{E}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G} \qquad \gamma_{zy} = \frac{\tau_{zy}}{G}$$

Exercise

Question A tensile test on a specimen with a cross section of 25 mm² gives the diagram on the right. The measurement base is (initially) $L_0 = 50$ mm long. Find the Young modulus E of the material.



Answer The elastic modulus is the slope of the linear part of the stress-strain diagram:

$$E = \left(\frac{\sigma}{\varepsilon}\right)_P = \left(\frac{\frac{F}{A}}{\frac{\Delta L}{L_0}}\right)_P = \frac{\frac{60}{25}}{\frac{0.2}{50}} = 600\,\mathrm{N/mm^2}$$

Physical dimensions

Quantity	Physical dimension	SI unit
E, G	FL^{-2}	Pa
ν	–	–

ADDITIONAL READINGS

Other useful material properties: ductility

Ductility: measure of the degree of plastic (irreversible) deformation occurred prior to fracture

A material that undergoes very little plastic deformation is brittle