BEAM THEORY – MATERIAL FAILURE THEORIES

STRUCTURAL MECHANICS

The ERAMCA Project

Environmental Risk Assessment and Mitigation on Cultural Heritage assets in Central Asia

v2022317

TABLE OF CONTENTS

Environmental Risk Assessment and Mitigation on Cultural Heritage Assets in Central Asia

LECTURER/STUDENTS OBJECTIVES

☞ Present the failure due to exceeding the strength limit in the cross sections and the concept of equivalent stress.

👥 Understand the concept of equivalent stress in relation to different materials and apply it to beams.

INTRODUCTION

ERAMCA

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Environmental Risk Assessment and Mitigation on Cultural
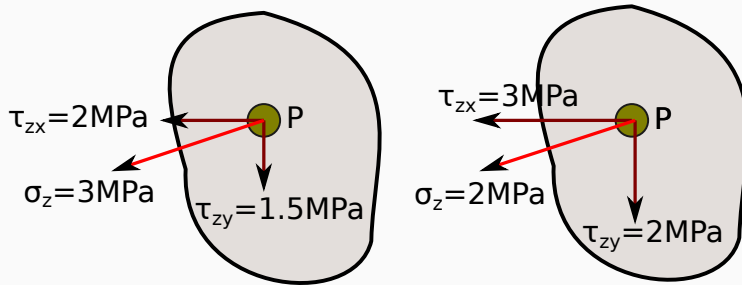Heritage Assets in Central Asia

Normal and tangential stresses

- What is the worse condition for a point P?
- How can we take into account all stresses at P to evaluate their "total" effect?

The material failure theory (or strength criteria) represents a way to predict the conditions under which solid materials fail under the action of external loads

The aim is...

...to find a index of the stress of the material, or equivalent stress $\sigma_{eq} = \sigma_{eq}(\sigma_1, \sigma_2)$, to compare with the strength limit values measured in a uniaxial laboratory test ($f_{r(T)}$ in tension and $f_{r(C)}$ in compression)

It is assumed that $f_{r(C)}, f_{p(C)}$ and $\sigma_{e(C)}$ are positive values

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- Ductile materials: the strength in tension $f_{r(T)}$ is usually equal to the strength in compression $f_{r(C)}$
- Brittle materials: the strength in tension $f_{r(T)}$ is usually different from the strength in compression $f_{r(C)}$
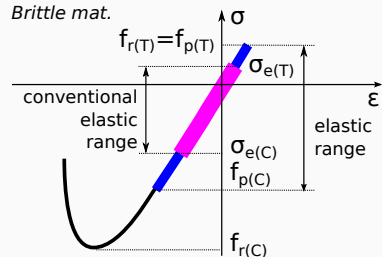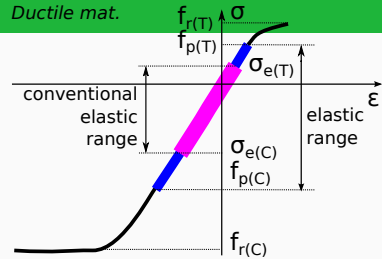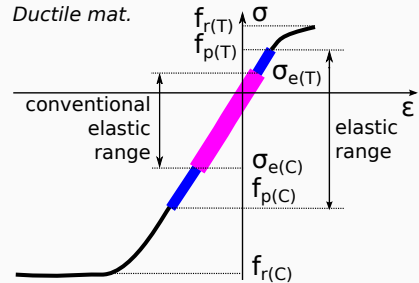
PROPORTIONAL LIMITS

The elastic range is defined by $f_{p(T)}$ and $f_{p(C)}$

To obtain a safety margin, $f_{p(T)}$ and $f_{p(C)}$ are reduced by partial safety factors $\gamma_{M(T)}$ and $\gamma_{M(C)}$

$$\sigma_{e(T)} = \frac{f_{p(T)}}{\gamma_{M(T)}} \quad \sigma_{e(C)} = \frac{f_{p(C)}}{\gamma_{M(C)}}$$

to define the conventional elastic range $(-\sigma_{e(C)}, \sigma_{e(T)})$

BRITTLE MATERIALS

This criterion requires to restrict the three principal stresses inside the the conventional elastic range $(-\sigma_{e(C)}, \sigma_{e(T)})$

$$-\sigma_{e(C)} \leq \sigma_1 \leq \sigma_{e(T)} \quad -\sigma_{e(C)} \leq \sigma_2 \leq \sigma_{e(T)} \quad -\sigma_{e(C)} \leq 0 \leq \sigma_{e(T)}$$

It is represented by a square in the space described by the principal stresses (σ_1, σ_2)

DUCTILE MATERIALS

This criterion requires to limit the maximum tangential stress τ_{max}:
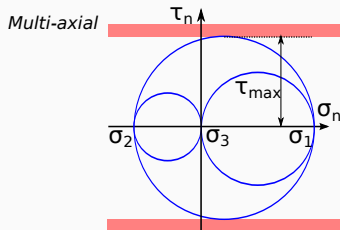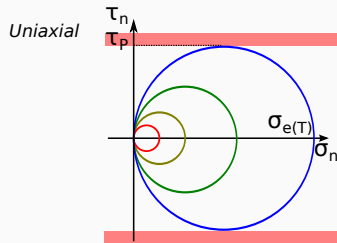
$$\tau_{max} \leq \tau_P = \frac{1}{2}\sigma_{e(T)}$$

where:

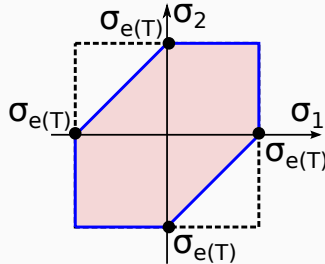$$\tau_{max} = \frac{1}{2}\max\left\{|\sigma_1 - \sigma_2|; |\sigma_1 - 0|; |\sigma_2 - 0|\right\}$$

It is obtained:

$$\sigma_{eq} = \max\left\{|\sigma_1 - \sigma_2|; |\sigma_1 - 0|; |\sigma_2 - 0|\right\} \leq \sigma_{e(T)}$$
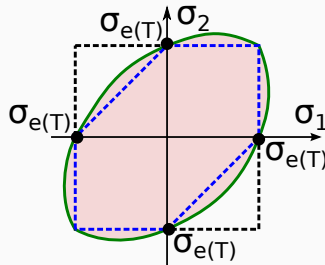
It is represented by an hexagon in the space described by the principal stresses (σ_1, σ_2)

It is represented by an ellipse in the space described by the principal stresses (σ_1, σ_2)

$$\sigma_{eq} = \sqrt{\sigma_1{}^2 + \sigma_2{}^2 - \sigma_1\sigma_2} \leq \sigma_{e(T)}$$
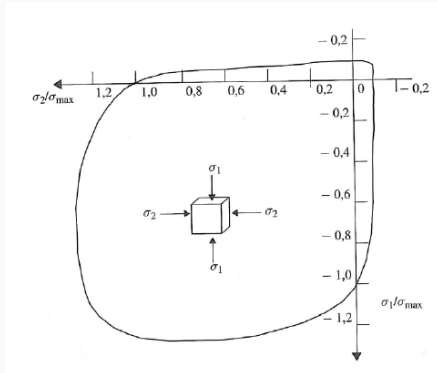
EXPERIMENTAL CONFIRMATION

H.B. Kupfer, K.H. Gerstle, *Behavior of Concrete under Biaxial Stresses*, Journal of the Engineering Mechanics Division, vol. 99, n. 4, 1973, pp. 853-866
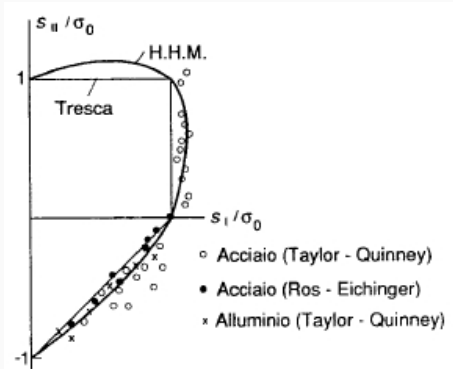
L. Corradi, *Meccanica delle Strutture*, vol. 1, McGraw Hill, 1992 (*acciaio*: steel; *alluminio*: aluminum; HHM: Huber-Hencky-von Mises criterion)

VERIFICATION CONDITIONS (DUCTILE MATERIALS)

The principal stresses are given by (Mohr's circle):

$$\sigma_1, \sigma_2 = \frac{\sigma_z}{2} \pm \frac{1}{2}\sqrt{\sigma_z^2 + 4\tau_z^2}; \quad \sigma_3 = 0$$

the equivalent stress by:

$$\sigma_{eq} = \max\{|\sigma_1 - \sigma_2|; |\sigma_1|; |\sigma_2|\} \quad \text{(Tresca criterion)}$$

$$\sigma_{eq} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \quad \text{(von Mises criterion)}$$

The verification condition is passed if:

$$\sigma_{eq} = \sqrt{\sigma_z^2 + \mathcal{D}\tau_z^2} \leq \frac{f_{p(T)}}{\gamma_{M(T)}} = \sigma_{e(T)}$$

where
$$\begin{cases} \mathcal{D} = 4 & \text{Tresca criterion} \\ \mathcal{D} = 3 & \text{von Mises criterion} \end{cases}$$

The study of structures under the elastic behavior can give unreliable results.

- The crack formations beyond the elastic phase does not means the loss of functionality of the structure. Sometimes, the fracture can be considered as an hinge, without loss of stability.
- Holes and other particular situations (notches, sharp angles) give stress concentrations (and/or infinite stresses) that can be avoided using elasto-plastic models (stress redistribution).