BEAM THEORY – MATERIAL FAILURE THEORIES

STRUCTURAL MECHANICS

The ERAMCA Project

Environmental Risk Assessment and Mitigation on Cultural Heritage assets in Central Asia

v2022317

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Lecturer/students objectives

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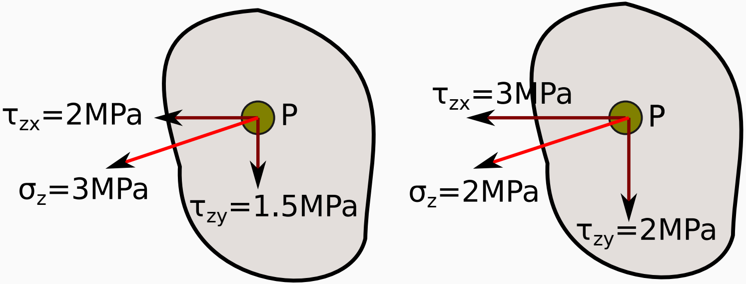
- Present the failure due to exceeding the strength limit in the cross sections and the concept of equivalent stress.
- Understand the concept of equivalent stress in relation to different materials and apply it to beams.

Introduction

Normal and tangential stresses

- What is the worse condition for a point P?
- How can we take into account all stresses at P to evaluate their "total" effect?



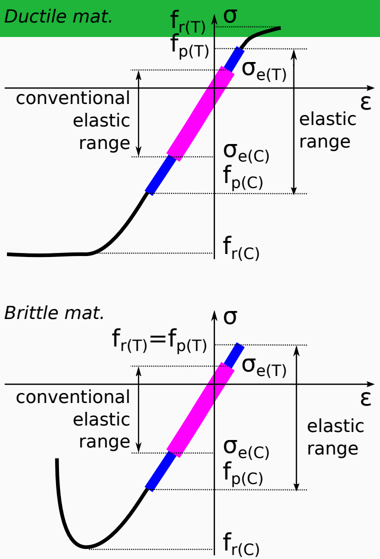
The material failure theory (or strength criteria) represents a way to predict the conditions under which solid materials fail under the action of external loads

The aim is…

…to find a index of the stress of the material, or equivalent stress $\sigma_{eq} = \sigma_{eq}(\sigma_1, \sigma_2)$, to compare with the strength limit values measured in a uniaxial laboratory test ($f_{r(T)}$ in tension and $f_{r(C)}$ in compression)

It is assumed that $f_{r(C)}$, $f_{p(C)}$ and $\sigma_{e(C)}$ are positive values

DUCTILE AND BRITTLE MATERIALS



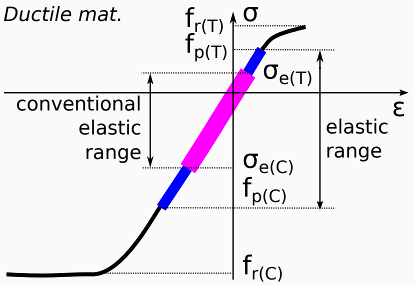
- Ductile materials: the strength in tension $f_{r(T)}$ is usually equal to the strength in compression $f_{r(C)}$
- Brittle materials: the strength in tension $f_{r(T)}$ is usually different from the strength in compression $f_{r(C)}$

The elastic range is defined by $f_{p(T)}$ and $f_{p(C)}$

To obtain a safety margin, $f_{p(T)}$ and $f_{p(C)}$ are reduced by partial safety factors $\gamma_{M(T)}$ and $\gamma_{M(C)}$

$$\sigma_{e(T)} = \frac{f_{p(T)}}{\gamma_{M(T)}} \quad \sigma_{e(C)} = \frac{f_{p(C)}}{\gamma_{M(C)}}$$

to define the conventional elastic range $(-\sigma_{e(C)}, \sigma_{e(T)})$



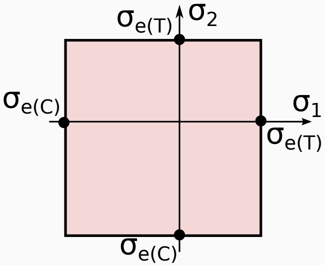
Brittle materials

Galileo-Rankine criterion

This criterion requires to restrict the three principal stresses inside the the conventional elastic range $(-\sigma_{e(C)}, \sigma_{e(T)})$

$$-\sigma_{e(C)} \leq \sigma_1 \leq \sigma_{e(T)} \quad -\sigma_{e(C)} \leq \sigma_2 \leq \sigma_{e(T)} \quad -\sigma_{e(C)} \leq 0 \leq \sigma_{e(T)}$$

It is represented by a square in the space described by the principal stresses (σ_1,σ_2)



Ductile materials

This criterion requires to limit the maximum tangential stress τ_{max}:

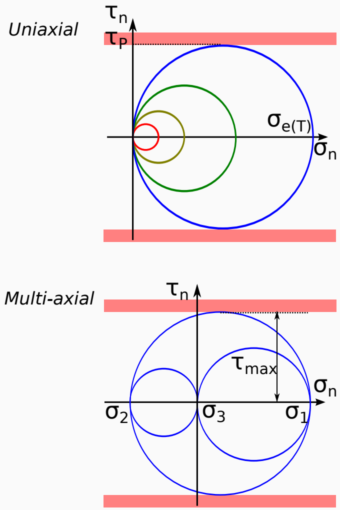
$$\tau_{max} \leq \tau_P = \frac{1}{2}\sigma_{e(T)}$$

where:

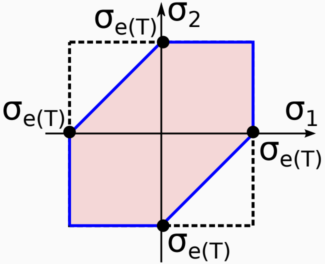
$$\tau_{max} = \frac{1}{2}\max\{|\sigma_1 - \sigma_2|; |\sigma_1 - 0|; |\sigma_2 - 0|\}$$

It is obtained:

$$\sigma_{eq} = \max\{|\sigma_1 - \sigma_2|; |\sigma_1 - 0|; |\sigma_2 - 0|\} \leq \sigma_{e(T)}$$

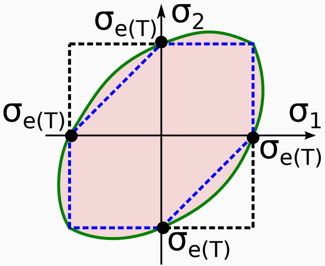


It is represented by an hexagon in the space described by the principal stresses (σ_1,σ_2)



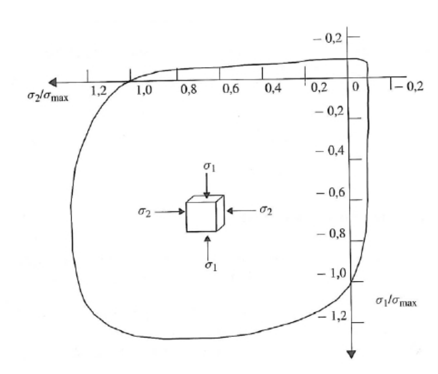
It is represented by an ellipse in the space described by the principal stresses (σ_1, σ_2)

$$\sigma_{eq} = \sqrt{{\sigma_1}^2 + {\sigma_2}^2 - \sigma_1 \sigma_2} \leq \sigma_{e(T)}$$

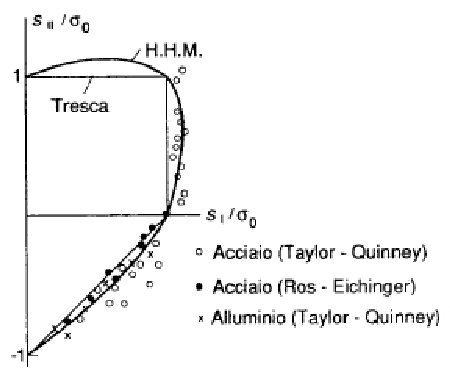


Experimental confirmation

Experiments (concrete, steel and aluminum)



H.B. Kupfer, K.H. Gerstle, *Behavior of Concrete under Biaxial Stresses*, Journal of the Engineering Mechanics Division, vol. 99, n. 4, 1973, pp. 853-866



L. Corradi, *Meccanica delle Strutture*, vol. 1, McGraw Hill, 1992 (*acciaio*: steel; *alluminio*: aluminum; HHM: Huber-Hencky-von Mises criterion)

Verification conditions (ductile materials)

The principal stresses are given by (Mohr's circle):

$$\sigma_1, \sigma_2 = \frac{\sigma_z}{2} \pm \frac{1}{2}\sqrt{{\sigma_z}^2 + 4\,{\tau_z}^2}; \quad \sigma_3 = 0$$

the equivalent stress by:

$$\sigma_{eq} = \max\{|\sigma_1 - \sigma_2|; |\sigma_1|; |\sigma_2|\} \quad \text{(Tresca criterion)}$$

$$\sigma_{eq} = \sqrt{{\sigma_1}^2 + {\sigma_2}^2 - \sigma_1\sigma_2} \quad \text{(von Mises criterion)}$$

The verification condition is passed if:

$$\sigma_{eq} = \sqrt{{\sigma_z}^2 + \mathcal{D}\,{\tau_z}^2} \leq \frac{f_{p(T)}}{\gamma_{M(T)}} = \sigma_{e(T)}$$

$$\text{where} \quad \begin{cases} \mathcal{D} = 4 & \text{Tresca criterion} \\ \mathcal{D} = 3 & \text{von Mises criterion} \end{cases}$$

Beyond elasticity...

The study of structures under the elastic behavior can give unreliable results.

- The crack formations beyond the elastic phase does not means the loss of functionality of the structure. Sometimes, the fracture can be considered as an hinge, without loss of stability.
- Holes and other particular situations (notches, sharp angles) give stress concentrations (and/or infinite stresses) that can be avoided using elasto-plastic models (stress redistribution).