BEAM THEORY – TANGENTIAL STRESS DUE TO SHEAR

STRUCTURAL MECHANICS

The ERAMCA Project

Environmental Risk Assessment and Mitigation on Cultural Heritage assets in Central Asia

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Environmental Risk Assessment and Mitigation on Cultural Heritage Assets in Central Asia

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Lecturer/students objectives

LECTURER/STUDENTS OBJECTIVES

- Present the tangential stress (shear) calculation shear for beams.
- Understand the hypotheses, distinguish the different loading condition and apply the proper solutions.

INTRODUCTION

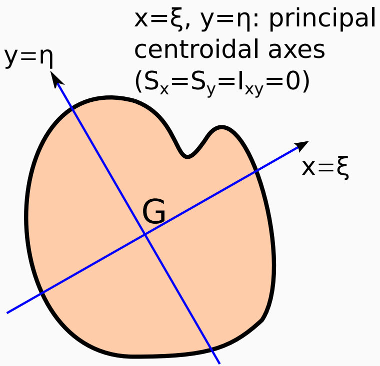
Throughout the slides...

...the principal centroidal axes, labeled as (x, y) instead of (ξ, η), are used!

It should be noted that...

... bending and shear are coupled. From beam theory, it is:

$$T_y(z) = \frac{dM_x(z)}{dz}$$



$$N_z = \int_A \sigma_z dA$$

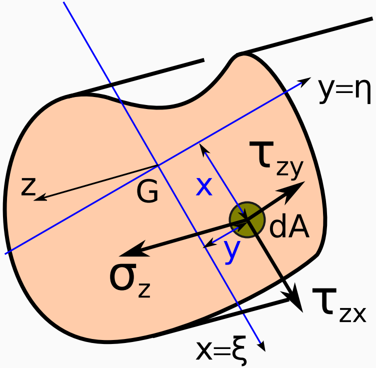
$$T_x = \int_A \tau_{zx} dA$$

$$T_y = \int_A \tau_{zy} dA$$

$$M_x = \int_A \sigma_z y dA$$

$$M_y = -\int_A \sigma_z x dA$$

$$M_z = \int_A (\tau_{zy} x - \tau_{zx} y) dA$$

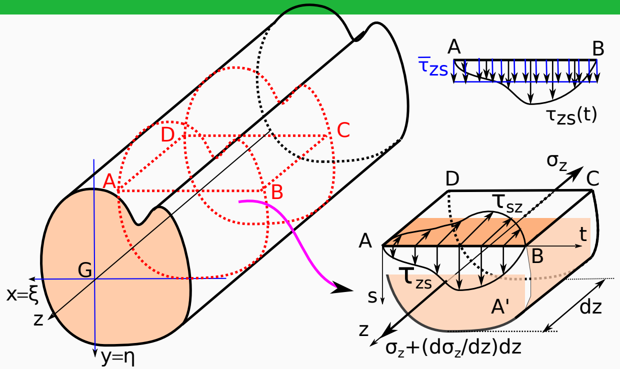


Bending and shear

Bending and shear

Bending, shear and twisting couple

NORMAL FORCE AND DIRECT SHEAR (RIVETS)



The equilibrium along z axis of the element of area A' gives:

$$\underbrace{\int_{A'}\left(-\sigma_z+\sigma_z+\frac{d\sigma_z}{dz}dz\right)dA'}_{\text{Integral \#1}}-\underbrace{\int_{ABCD}\tau_{sz}(t)\,dt\,dz}_{\text{Integral \#2}}=0$$

Integral #1, taking into account that shear is always present together with bending moment (recall that $T_y = \frac{dM_x}{dz}$):

$$\frac{d\sigma_z}{dz} = \frac{d}{dz}\left(\frac{M_x y}{I_x}\right) = \frac{y}{I_x}\frac{dM_x}{dz} = \frac{y}{I_x}T_y$$

$$\int_{A'}\left(\frac{y}{I_x}T_y\right) dz\, dA' = \frac{T_y}{I_x}dz \int_{A'} y dA' = \frac{T_y}{I_x}dz\, S_x^{A'}$$

Integral #2 ($\overline{\tau}_{sz}$ is the average value along t!):

$$\int_{ABCD} \tau_{sz}(t)\, dt\, dz = \overline{\tau}_{sz} dz \int_{AB} dt = \overline{\tau}_{sz}\, dz\, \overline{AB}$$

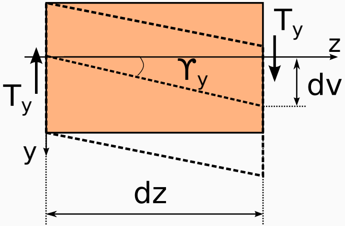
and, dividing by dz:

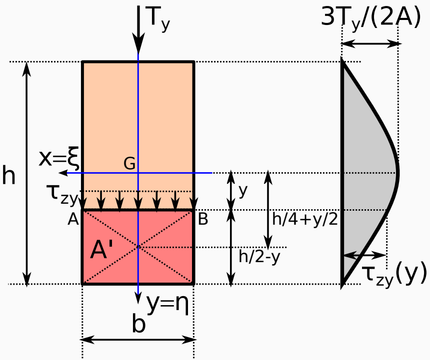
$$\overline{\tau}_{sz} = \frac{T_y\, S_x^{A'}}{I_x\, \overline{AB}}$$ (Jourawski o Žuravskij's formula)

Deformation due to shear

$$\gamma_y = t_y \frac{T_y}{G\,A}$$

Coefficient t_y depends upon the shape of the cross section (6/5 for a rectangular cross section and 4/3 for a circular cross section)





The first moment of the area $A' = b\left(\frac{h}{2} - y\right)$ is:

$$S_x^{A'} = +A'\left(\frac{h}{4} + \frac{y}{2}\right) = +b\left(\frac{h}{2} - y\right)\left(\frac{h}{4} + \frac{y}{2}\right) = +\frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$$

a parabolic shape of the stress distribution is obtained:

$$\overline{\tau}_{yz}(y) = \frac{T_y S_x^{A'}}{I_x \overline{AB}} = \frac{T_y \left[+\frac{b}{2}\left(\frac{h^2}{4} - y^2\right)\right]}{\left(\frac{bh^3}{12}\right)(b)} = +\frac{6T_y}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

The maximum value is reached for $y = 0$:

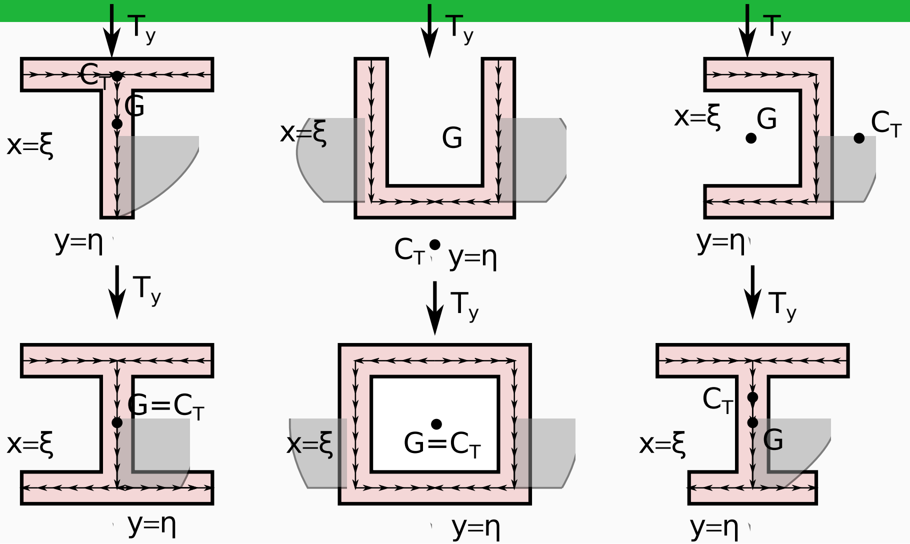
$$\overline{\tau}_{yz,max} = \overline{\tau}_{yz}(y = 0) = \frac{3}{2}\frac{T_y}{bh} = \frac{3}{2}\frac{T_y}{A}$$

while, for $y = \pm\frac{h}{2}$ the shear stresses are equal to zero.

The positive sign of shear stress...

... means stresses directed toward the area A' through the line AB

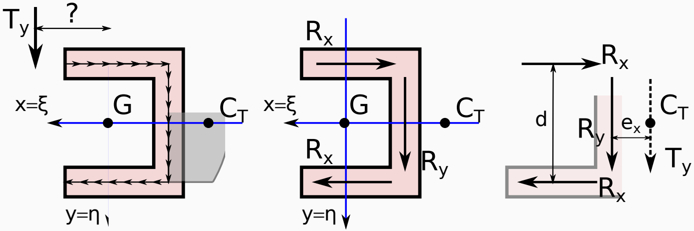
SHAPE OF THE SHEAR STRESSES DIAGRAM



Shear center

The line of action of the resultant of the shear stresses does not pass through the centroid G but through the shear center C_T

- Equivalence in the horizontal direction: verified
- Equivalence in the vertical direction $\Longrightarrow R_y = T_y$
- Equivalence in terms of moments $T_y e_x = R_x d + R_y 0 \Longrightarrow e_x = \frac{R_x d}{T_y}$



It is noticed that a force...

... applied to the shear center gives bending without torque

Shear center – experiments (2/5)

SHEAR CENTER – EXPERIMENTS (3/5)

SHEAR CENTER – EXPERIMENTS (4/5)

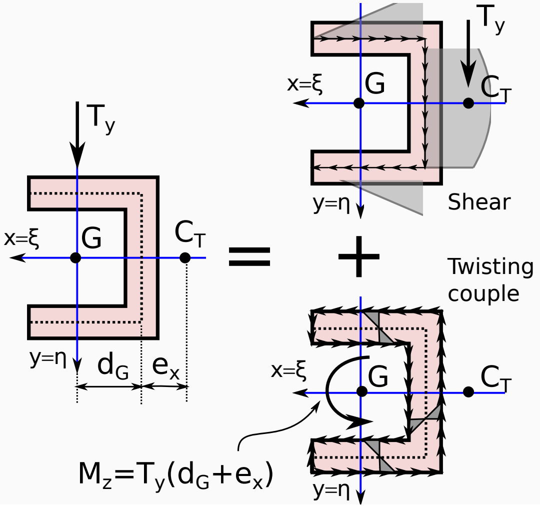
SHEAR CENTER – EXPERIMENTS

The stresses due to a shear force applied on a line not passing through the shear center C_T is calculated adding, for each point, the stress due to:

- shear T_y (Jourawski's formula)
- twisting couple $M_{z,C_T} = T_y(d_G + e_x)$, calculated with respect to the shear center C_T

In the example above...

... the stresses due to M_{z,C_T} will be calculated with the equations valid for the thin-walled open members (see the slides about stresses due to twisting couple)


T
y
x=ξ
G
C
T
y=η
d
G
e
x
=
+
Shear
Twisting
couple
M
z
=T
y
(d
G
+e
x
)

Note

- Shear forces T_j are positive when directed along the positive direction of the axis j (j is one of the principal axis x, y).
- The sign of shear stresses is determined by the sign of the first moment of the area $S_j^{A'}$ and from the sign of T_j. They are positive if directed toward the area A'.