

BEAM THEORY – NOTES ON INTERNAL FORCES DIAGRAMS

STRUCTURAL MECHANICS

The ERAMCA Project

Environmental Risk Assessment and Mitigation on Cultural Heritage assets in Central Asia

v2022317

This work is licensed under a [Creative Commons "Attribution-ShareAlike 4.0 International"](https://creativecommons.org/licenses/by-sa/4.0/) license.



TABLE OF CONTENTS



Lecturer/students objectives

Some rules

Simply supported beam

Cantilever

LECTURER/STUDENTS OBJECTIVES

-  Present diagram for simple beams.
-  Understand the relations between internal forces, loads and boundary conditions to acquire a critical view.

SOME RULES

- Some simple schemes useful to understand how to plot diagrams of N , T and M are presented
- The following relationships hold:

$$\frac{dN(z)}{dz} = -p(z)$$

$$\frac{dT(z)}{dz} = -q(z)$$

$$\frac{dM(z)}{dz} = T(z)$$

Differentiating twice, it is obtained:

$$\frac{d^2M(z)}{dz^2} = \frac{dT(z)}{dz} = -q(z)$$

Some rules

- The diagrams must be compatible with external constraints (i.e., moment equal to zero in the hinges or at free ends, shear equal to zero at free ends...)
- If shear $T(z)$ is zero, moment $M(z)$ presents a maximum or a minimum
- If $q(z)$ is zero, shear $T(z)$ presents maximum or a minimum
- If $p(z)$ is zero, axial force $N(z)$ presents a maximum or a minimum
- The second derivative of $M(z)$, i.e., the concavity, is $-q(z)$

Some rules

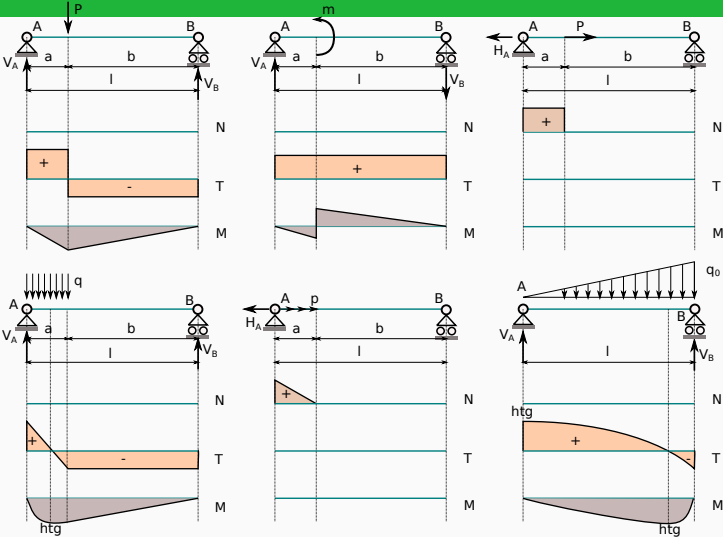
- A concentrated force orthogonal to the beam longitudinal axis gives a discontinuity of $T(z)$ and a cusp of $M(z)$
- A concentrated force parallel to the beam longitudinal axis gives a discontinuity of $N(z)$
- A couple gives a discontinuity of $M(z)$

Some rules

- If the load $q(z)$ is **equal to zero**, shear $T(z)$ is piecewise constant and moment $M(z)$ linear (polynomial function of the first degree)
- If the load $q(z)$ is **constant**, shear $T(z)$ is piecewise linear and moment $M(z)$ parabolic (polynomial function of the second degree)
- If the load $q(z)$ is **linear**, shear $T(z)$ is parabolic and moment $M(z)$ a polynomial function of the third degree

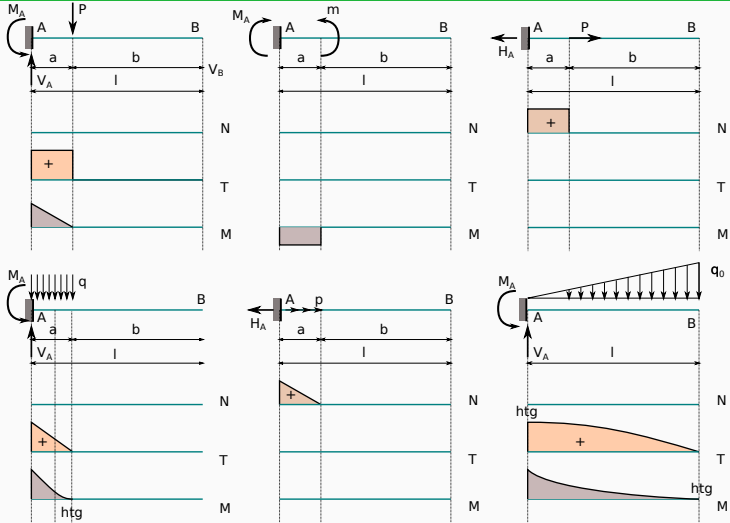
SIMPLY SUPPORTED BEAM

SIMPLY SUPPORTED BEAM



CANTILEVER

CANTILEVER



Signs

The sign conventions for axial force $N(z)$, shear force $T(z)$ and bending moment $M(z)$ are:

- axial force positive if tensile
- shear positive if it causes a clockwise rotation of the beam element
- moment $M(z)$ must be plotted on the tension side of the beam (if it is on the right side with respect to the positive direction of z , otherwise is negative)

Horizontal tangent

The points where the tangent to a diagram is horizontal are indicated by the label **htg**