BEAM THEORY – NOTES ON INTERNAL FORCES DIAGRAMS

STRUCTURAL MECHANICS

The ERAMCA Project

Environmental Risk Assessment and Mitigation on Cultural Heritage assets in Central Asia

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ERAMCA
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Environmental Risk Assessment and Mitigation on Cultural Heritage Assets in Central Asia

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Lecturer/students objectives

LECTURER/STUDENTS OBJECTIVES

- Present diagram for simple beams.
- Understand the relations between internal forces, loads and boundary conditions to acquire a critical view.

Some rules

- Some simple schemes useful to understand how to plot diagrams of N, T and M are presented
- The following relationships hold:

$$\begin{aligned} \frac{\mathrm{d}N(z)}{\mathrm{d}z} &= -p(z) \\ \frac{\mathrm{d}T(z)}{\mathrm{d}z} &= -q(z) \\ \frac{\mathrm{d}M(z)}{\mathrm{d}z} &= T(z) \end{aligned}$$

Differentiating twice, it is obtained:

$$\frac{\mathrm{d}^2M(z)}{\mathrm{d}z^2} = \frac{\mathrm{d}T(z)}{\mathrm{d}z} = -q(z)$$

Some rules

- The diagrams must be compatible with external constraints (i.e., moment equal to zero in the hinges or at free ends, shear equal to zero at free ends...)
- If shear $T(z)$ is zero, moment $M(z)$ presents a maximum or a minimum
- If $q(z)$ is zero, shear $T(z)$ presents maximum or a minimum
- If $p(z)$ is zero, axial force $N(z)$ presents a maximum or a minimum
- The second derivative of $M(z)$, i.e., the concavity, is $-q(z)$

Some rules

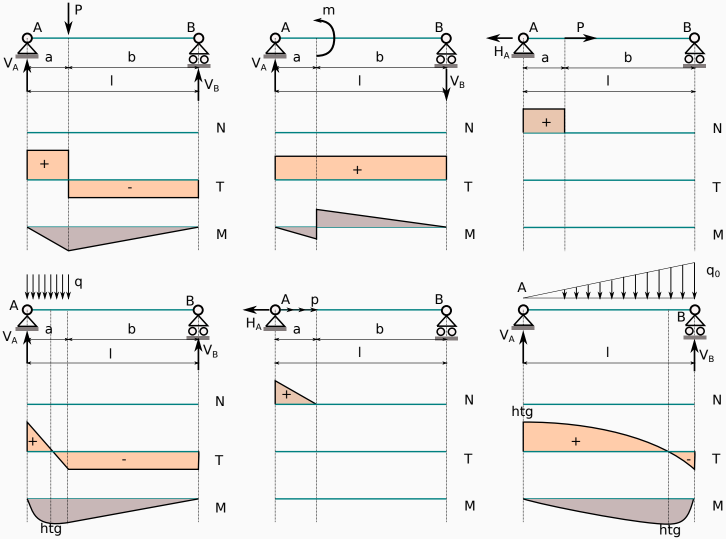
- A concentrated force orthogonal to the beam longitudinal axis gives a discontinuity of $T(z)$ and a cusp of $M(z)$
- A concentrated force parallel to the beam longitudinal axis gives a discontinuity of $N(z)$
- A couple gives a discontinuity of $M(z)$

Some rules

- If the load $q(z)$ is equal to zero, shear $T(z)$ is piecewise constant and moment $M(z)$ linear (polynomial function of the first degree)
- If the load $q(z)$ is constant, shear $T(z)$ is piecewise linear and moment $M(z)$ parabolic (polynomial function of the second degree)
- If the load $q(z)$ is linear, shear $T(z)$ is parabolic and moment $M(z)$ a polynomial function of the third degree

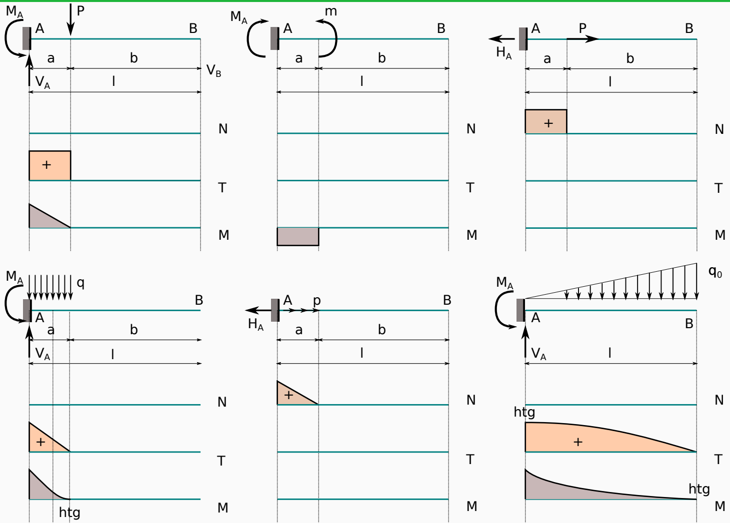
Simply supported beam

SIMPLY SUPPORTED BEAM



Cantilever

CANTILEVER



M_A
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b
V_B
V_A
l
N
+
T
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Sign conventions

Signs

The sign conventions for axial force $N(z)$, shear force $T(z)$ and bending moment $M(z)$ are:

- axial force positive if tensile
- shear positive if it causes a clockwise rotation of the beam element
- moment $M(z)$ must be plotted on the tension side of the beam (if it is on the right side with respect to the positive direction of z, otherwise is negative)

Horizontal tangent

The points where the tangent to a diagram is horizontal are indicated by the label htg