BEAM THEORY – STATICS

STRUCTURAL MECHANICS

The ERAMCA Project

Environmental Risk Assessment and Mitigation on Cultural Heritage assets in Central Asia

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Lecturer/students objectives

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- Present the main aspects of the static of beams subjected to different actions.
- Understand the relationships between loads and internal forces, apply the equilibrium equations to solve statically determinate structures.

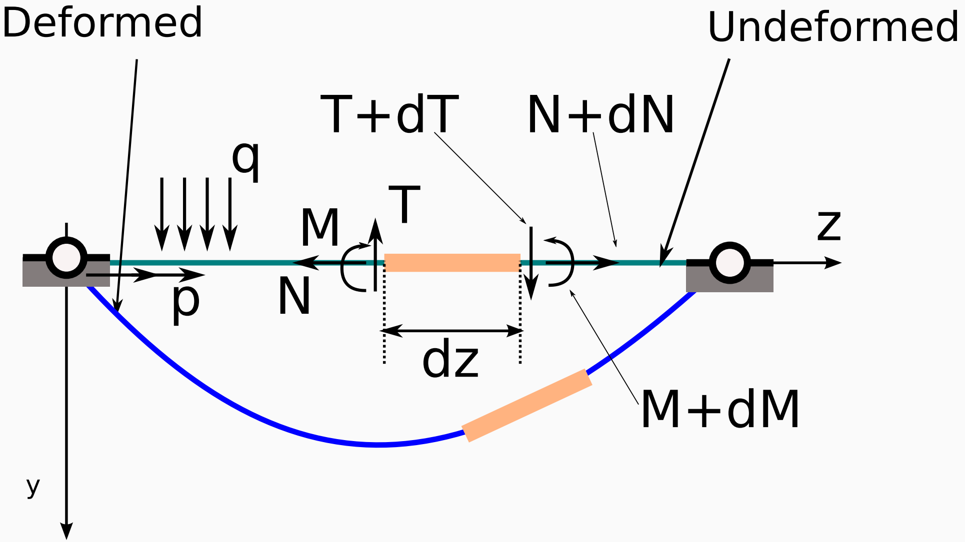
Introduction

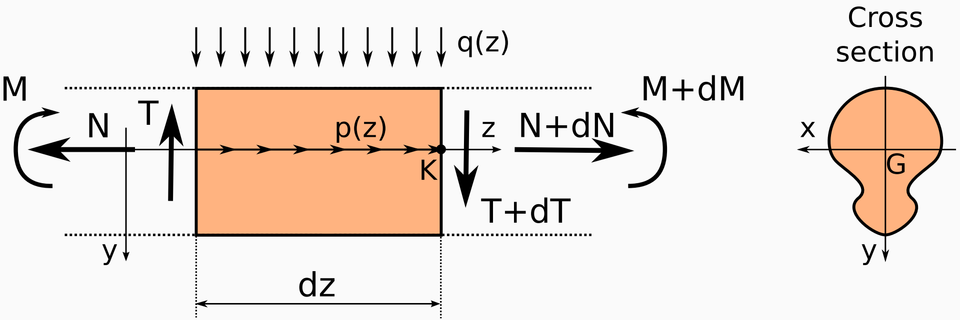
Aim of the lesson

The aim of the lecture is to provide the information necessary to understand the relations between external loads and internal forces, i.e., equilibrium equations

Equilibrium

Equilibrium of the beam




q(z)
M
T
N
p(z)
z
N+dN
M+dM
K
T+dT
y
dz
Cross
section
x
G
y

External loads on a elemental portion of beam of length dz:

- $q(z)$: distributed load perpendicular to the z axis of the beam
- $p(z)$: distributed load parallel to the z axis of the beam

so that:

- $q(z)\mathrm{d}z$: external force perpendicular to the axis z of the beam
- $p(z)\mathrm{d}z$: external force parallel to the axis z of the beam

Internal reaction forces on a portion of beam of length dz:

- left-hand side (lhs) end of the element:
  - $N(z)$: axial force
  - $T(z)$: shear force
  - $M(z)$: bending moment
- right-hand side (rhs) end of the element:
  - $N(z) + \mathrm{d}N(z)$: axial force
  - $T(z) + \mathrm{d}T(z)$: shear force
  - $M(z) + \mathrm{d}M(z)$: bending moment

Notice that…

… on the rhs the internal forces are different (increased or decreased) with respect to the ones acting on the lhs

- Equilibrium along z axis:

$$-N(z) + N(z) + \mathrm{d}N(z) + p(z)\mathrm{d}z = 0 \quad \text{i.e.,} \quad \frac{\mathrm{d}N(z)}{\mathrm{d}z} = -p(z)$$

- Equilibrium along y axis:

$$-T(z) + T(z) + \mathrm{d}T(z) + q(z)\mathrm{d}z = 0 \quad \text{i.e.,} \quad \frac{\mathrm{d}T(z)}{\mathrm{d}z} = -q(z)$$

- Equilibrium about K point:

$$-M(z) + M(z) + \mathrm{d}M(z) - T(z)\mathrm{d}z + \underbrace{[q(z)\mathrm{d}z]\left(\frac{\mathrm{d}z}{2}\right)}_{\text{Neglected}} = 0$$

$$\text{so that} \quad \frac{\mathrm{d}M(z)}{\mathrm{d}z} = T(z)$$

Finally:

$$\frac{\mathrm{d}N(z)}{\mathrm{d}z} = -p(z)$$

$$\frac{\mathrm{d}T(z)}{\mathrm{d}z} = -q(z)$$

$$\frac{\mathrm{d}M(z)}{\mathrm{d}z} = T(z)$$

Moreover, it can be useful to obtain, by differentiating:

$$\frac{\mathrm{d}^2 M(z)}{\mathrm{d}z^2} = \frac{\mathrm{d}T(z)}{\mathrm{d}z} = -q(z)$$

According to the reference frame, the notation means:

$$N \equiv N_z, \quad T \equiv T_y, \quad M \equiv M_x$$

Physical dimensions

Quantity	Physical dimension	SI unit
N, T	F	N
M	FL	N m
q, p	FL^{-1}	N/m

Sign conventions

Signs

- q, p: positive if directed along the positive direction of (y, z), respectively
- N: positive if causes tension
- T: positive if causes a clockwise rotation of the beam
- M: plotted on the tension side of the beam