BEAM THEORY – STRESS ANALYSIS

STRUCTURAL MECHANICS

The ERAMCA Project

Environmental Risk Assessment and Mitigation on Cultural Heritage assets in Central Asia

v2022317

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Lecturer/students objectives

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- Describe the internal forces exerted by neighbouring particles and present the fundamental concept of stress for beams.
- Understand the equilibrium of beams by means of internal surface forces (stress).

Introduction

Stress definition

Stress

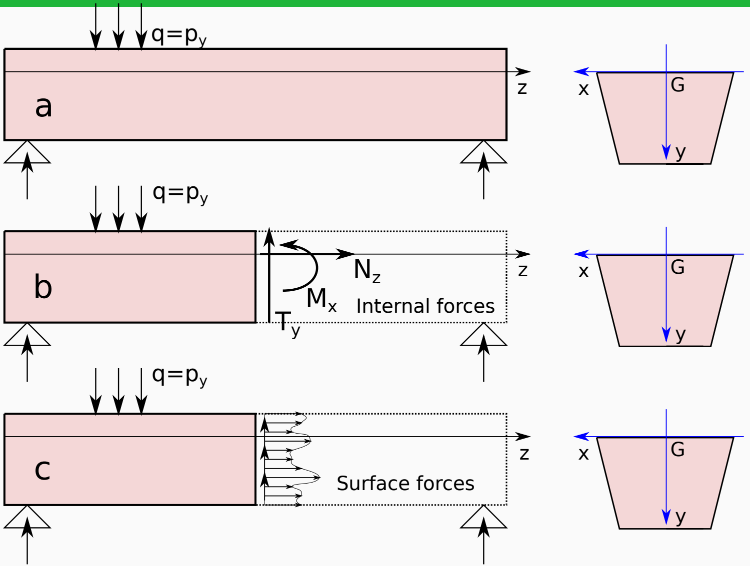
The mathematical description of equilibrium conditions lead to the idea of stress.

- The concept of stress is fundamental for the study of continuum mechanics
- The first contributions are by A.L. Cauchy who formulated a theory in 1822, based on the idea of pressure in fluids

Equilibrium of a plane beam

A beam or a structure must be:

a. globally in equilibrium: the external loads and the reactions from support must be a system of forces in equilibrium
b. in equilibrium for every possible part: the external loads and the internal forces N_z, T_y and M_x (that represent global forces on the section of the cut) must be a system of forces in equilibrium
c. in equilibrium for every possible part: the external loads and the stresses (surface forces acting on every point on the section of the cut) must be a system of forces in equilibrium

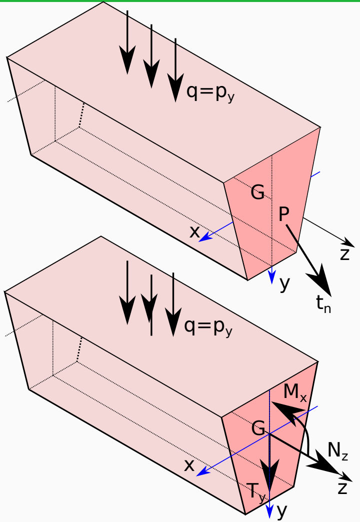


q=p$_y$
a
z
x
G
y
q=p$_y$
b
N$_z$
M$_x$
T$_y$
Internal forces
z
x
G
y
q=p$_y$
c
Surface forces
z
x
G
y

Equilibrium of a portion of the beam: stress

Surface forces called stresses ($\boldsymbol{t_n}$) must be present on the cut face to reach equilibrium

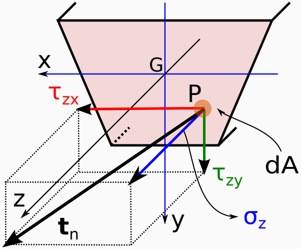
The resultant of these surface forces must be equivalent to the internal forces

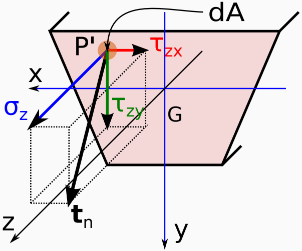


The components of the stress vector $\boldsymbol{t_n}$ acting on a point P of the cross section of the beam are three: τ_{zx}, τ_{zy} (shear or tangential stress) and $\sigma_{zz} = \sigma_z$ (normal stress)

The first subscript refers to the z axis orthogonal to the cross section, the second the direction of the stress

For other points (P') of the same cross section, vector $\boldsymbol{t_n}$ may have different magnitude and direction, i.e., $\boldsymbol{t_n} = \boldsymbol{t_n}(P) = \boldsymbol{t_n}(x, y)$





What do internal forces represent?

The internal forces are the resultant of the...

... stress vector $\boldsymbol{t_n}$ components σ_z, τ_{zx} and τ_{zy}

$$N_z = \int_A \sigma_z \, \mathrm{d}A \qquad T_x = \int_A \tau_{zx} \, \mathrm{d}A$$

$$M_x = \int_A \sigma_z y \, \mathrm{d}A \qquad T_y = \int_A \tau_{zy} \, \mathrm{d}A$$

$$M_y = -\int_A \sigma_z x \, \mathrm{d}A \qquad M_z = \int_A (\tau_{zy} x - \tau_{zx} y) \, \mathrm{d}A$$

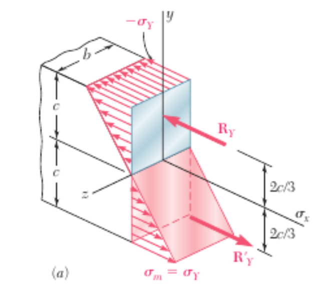
For a plane structure (or for symmetrical forces) the internal forces are three (i.e., N_z, T_y, M_x), otherwise they are six

Note that integration $\left(\int_A\right)$ is a continuous sum; summation $\left(\sum_{i=1}^{N}\right)$ is a discrete sum!

If the beam is statically determinate, N_z, T_y e M_x are found by using equilibrium equation only; if the beam is statically indeterminate by using compatibility equations

To find the stresses from the previous equations…

… their distribution over the area A of the cross section has to be known (wait next lectures!)



Sign conventions for stress

- Positive stress on positive faces (z axis pointing out) if in the positive direction of axes
- Negative stress on negative faces (z axis pointing in) if in the negative direction of axes

A (principal) reference system exists where…

… the stresses are described by σ_1, σ_2 e σ_3 only. The faces where these stresses act, the tangential stresses are zero

One principal stress is always zero for beams

Graphical procedure to find principal stresses and principal directions: Mohr's circles

- Equilibrium along z axis:

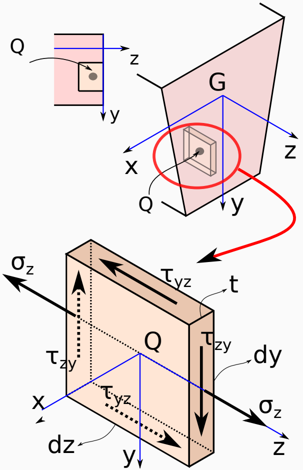
$$\sigma_z(\mathrm{d}y\ t) - \tau_{yz}(\mathrm{d}z\ t) + \tau_{yz}(\mathrm{d}z\ t) + \\ -\sigma_z(\mathrm{d}y\ t) = 0 \quad \text{OK!}$$

- Equilibrium along y axis:

$$-\tau_{zy}(\mathrm{d}y\ t) + \tau_{zy}(\mathrm{d}y\ t) = 0 \quad \text{OK!}$$

- Equilibrium about x axis:

$$\tau_{yz}(\mathrm{d}z\ t)\left(\frac{\mathrm{d}y}{2}\right) + \tau_{yz}(\mathrm{d}z\ t)\left(\frac{\mathrm{d}y}{2}\right) \\ -\tau_{zy}(\mathrm{d}y\ t)\left(\frac{\mathrm{d}z}{2}\right) - \tau_{zy}(\mathrm{d}y\ t)\left(\frac{\mathrm{d}z}{2}\right) = 0 \quad \text{i.e.,} \quad \tau_{zy} = \tau_{yz}$$



Physical dimensions

Parameter	Physical dimensions	SI unit
Stress σ_z, τ_{zx}, τ_{zy}	FL^{-2}	Pa
Area A, $\mathrm{d}\Omega_n$, $\mathrm{d}\Omega_y$,… $\mathrm{d}\Omega_z$	L^2	m^2
q, p_y	FL^{-1}	N/m