# **BEAM THEORY – STRESS ANALYSIS**

STRUCTURAL MECHANICS

The ERAMCA Project

#### Environmental Risk Assessment and Mitigation on Cultural Heritage assets in Central Asia

V2O22317

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Lecturer/students objectives

Introduction

**Stress definition** 





# **LECTURER/STUDENTS OBJECTIVES**





- Describe the internal forces exerted by neighbouring particles and present the fundamental concept of stress for beams.
- Understand the equilibrium of beams by means of internal surface forces (stress).





# **INTRODUCTION**





# **STRESS DEFINITION**





The mathematical description of equilibrium conditions lead to the idea of stress.

- The concept of stress is fundamental for the study of continuum mechanics
- The first contributions are by A.L. Cauchy who formulated a theory in 1822, based on the idea of pressure in fluids





(1/2)

A beam or a structure must be:

- a globally in equilibrium: the external loads and the reactions from support must be a system of forces in equilibrium
- b in equilibrium for every possible part: the external loads and the internal forces  $N_z$ ,  $T_y$  and  $M_x$  (that represent global forces on the section of the cut) must be a system of forces in equilibrium
- c in equilibrium for every possible part: the external loads and the stresses (surface forces acting on every point on the section of the cut) must be a system of forces in equilibrium



#### **EQUILIBRIUM OF A PLANE BEAM**



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#### **EQUILIBRIUM OF A PORTION OF THE BEAM: STRESS**

Surface forces called stresses  $(t_n)$  must be present on the cut face to reach equilibrium

The resultant of these surface forces must be equivalent to the internal forces







The components of the stress vector  $t_n$  acting on a point P of the cross section of the beam are three:  $\tau_{zx}$ ,  $\tau_{zy}$  (shear or tangential stress) and  $\sigma_{zz} = \sigma_z$  (normal stress)

The first subscript refers to the *z* axis orthogonal to the cross section, the second the direction of the stress

For other points (P') of the same cross section, vector  $\mathbf{t}_n$  may have different magnitude and direction, i.e.,  $\mathbf{t}_n = \mathbf{t}_n(P) = \mathbf{t}_n(x, y)$ 





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#### WHAT DO INTERNAL FORCES REPRESENT?

### The internal forces are the resultant of the...

... stress vector  $\mathbf{t}_n$  components  $\sigma_z$ ,  $\tau_{zx}$  and  $\tau_{zy}$ 

$$N_{z} = \int_{A} \sigma_{z} \, dA \qquad T_{x} = \int_{A} \tau_{zx} \, dA$$
$$M_{x} = \int_{A} \sigma_{z} y \, dA \qquad T_{y} = \int_{A} \tau_{zy} \, dA$$
$$M_{y} = -\int_{A} \sigma_{z} x \, dA \qquad M_{z} = \int_{A} (\tau_{zy} x - \tau_{zx} y) \, dA$$

For a plane structure (or for symmetrical forces) the internal forces are three (i.e.,  $N_z$ ,  $T_y$ ,  $M_x$ ), otherwise they are six



If the beam is statically determinate,  $N_z$ ,  $T_y \in M_x$  are found by using equilibrium equation only; if the beam is statically indeterminate by using compatibility equations

### To find the stresses from the previous equations...

... their distribution over the area A of the cross section has to be known (wait next lectures!)





- Positive stress on positive faces (*z* axis pointing out) if in the positive direction of axes
- Negative stress on negative faces (*z* axis pointing in) if in the negative direction of axes





## A (principal) reference system exists where..

... the stresses are described by  $\sigma_1$ ,  $\sigma_2 \in \sigma_3$  only. The faces where these stresses act, the tangential stresses are zero

One principal stress is always zero for beams

Graphical procedure to find principal stresses and principal directions: Mohr's circles



### **RECIPROCITY OF TANGENTIAL STRESS**

• Equilibrium along *z* axis:

$$\sigma_{z}(\mathrm{d} y \ t) - au_{yz}(\mathrm{d} z \ t) + au_{yz}(\mathrm{d} z \ t) + \ - \sigma_{z}(\mathrm{d} y \ t) = \mathrm{o} \quad \mathrm{OK!}$$

• Equilibrium along y axis:

$$- au_{zy}(\mathrm{d} y \ t) + au_{zy}(\mathrm{d} y \ t) = 0$$
 OK!

• Equilibrium about *x* axis:

$$\begin{aligned} \tau_{yz}(\mathrm{d}z \ t) \left(\frac{\mathrm{d}y}{2}\right) + \tau_{yz}(\mathrm{d}z \ t) \left(\frac{\mathrm{d}y}{2}\right) \\ -\tau_{zy}(\mathrm{d}y \ t) \left(\frac{\mathrm{d}z}{2}\right) - \tau_{zy}(\mathrm{d}y \ t) \left(\frac{\mathrm{d}z}{2}\right) = 0 \quad \text{i.e.,} \quad \tau_{zy} = 0 \end{aligned}$$

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Parameter	Physical dimensions	SI unit
Stress $\sigma_z$ , $ au_{zx}$ , $ au_{zy}$	$FL^{-2}$	Pa
Area A, $d\Omega_n$ , $d\Omega_y$ , $d\Omega_z$	L <sup>2</sup>	m²
<i>q</i> , <i>p</i> <sub>y</sub>	FL <sup>-1</sup>	N/m

