BEAM THEORY – STRUCTURAL SUPPORTS

STRUCTURAL MECHANICS

The ERAMCA Project

Environmental Risk Assessment and Mitigation on Cultural Heritage assets in Central Asia

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ERAMCA

Co-funded by the Erasmus+ Programme of the European Union

Environmental Risk Assessment and Mitigation on Cultural Heritage Assets in Central Asia

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LECTURER/STUDENTS OBJECTIVES

Lecturer/students objectives

- Present the most common supports in terms of displacements and forces.
- Understand how to calculate internal and external reactions using different supports and recognize (simple) unstable structures.

Introduction

The first step in the solution of any problem concerning the equilibrium of a rigid body is to construct an appropriate free-body diagram.

Free-body diagrams

It is necessary to show on the diagram the reactions through which the ground and other bodies oppose a possible motion of the body.

The idealized structural scheme of a real structure is kept fixed and in equilibrium by different structural supports.

Idealized structure

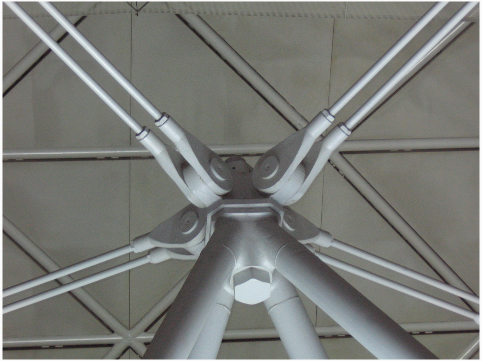
Main aspects to model a structure:

- geometry
- materials
- supports
- loads
- …

SUPPORTS

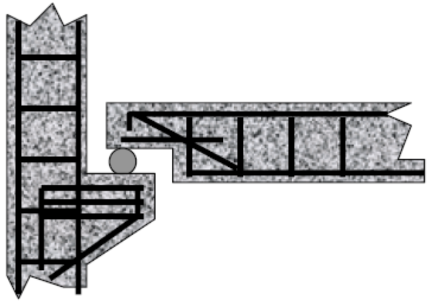
Real world supports (1/3)

Real world supports (2/3)

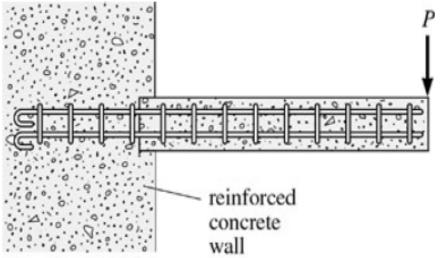


REAL WORLD SUPPORTS

Some schematic – concrete

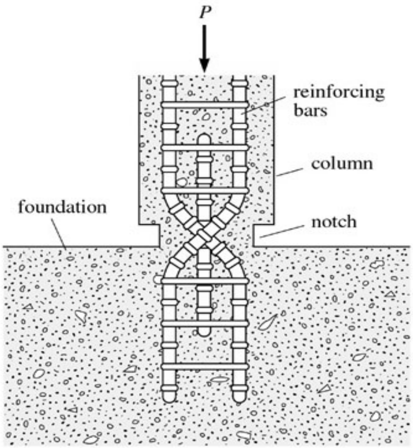


typical "roller-supported" connection (concrete)

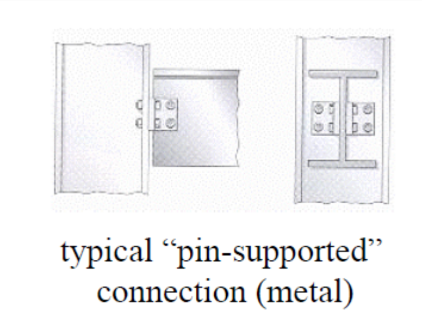


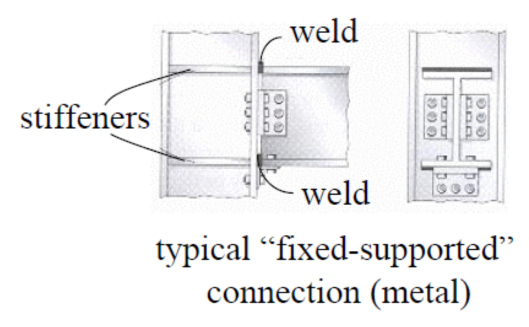
only beam reinforcement shown

SOME SCHEMATIC – CONCRETE

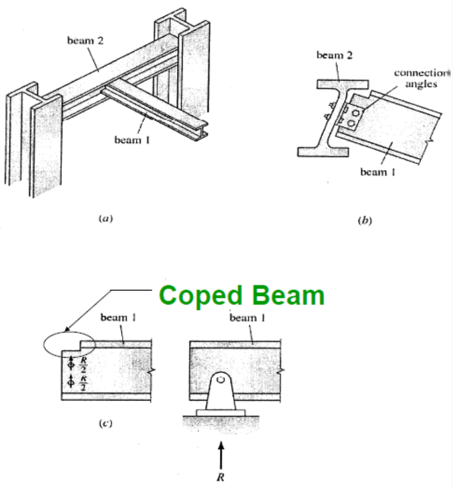


(b)

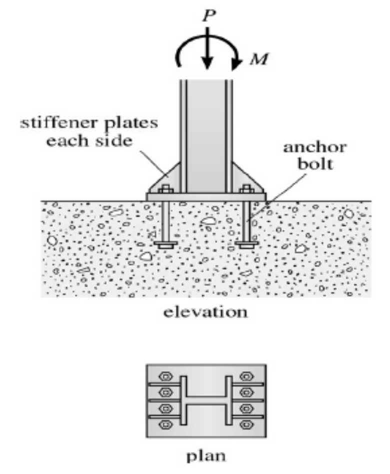

typical "pin-supported"
connection (metal)


weld
stiffeners
weld
typical "fixed-supported"
connection (metal)

SOME SCHEMATIC – STEEL



SOME SCHEMATIC – STEEL

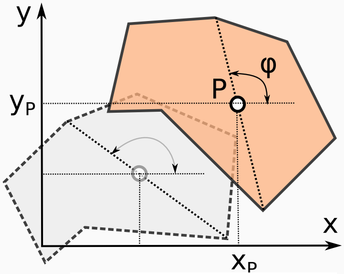


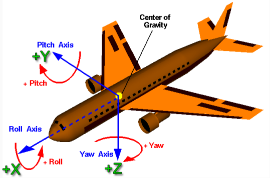
Classification

Degrees of freedom (DOF): number of independent parameters that define the configuration of a system

The position and orientation of a rigid body...

- ...in a plane is defined by two component of translation and one of rotation (i.e., three DOF, $g = 3$)
- ...in space is defined by three components of translation and three components of rotation (i.e., six DOF, $g = 6$)





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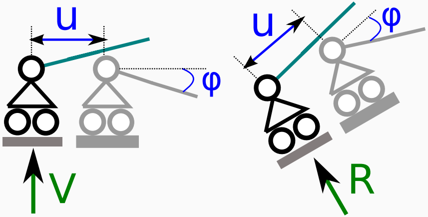
SUPPORTS – CLASSIFICATION

- Type of supports:
  - external supports
  - internal links
- Type of effect:
  - reactions (internal or external)
  - displacements (free or constrained)
- Degree of freedom:
  - number v of DOF constrained
- Reactions:
  - number and type of reactions provided

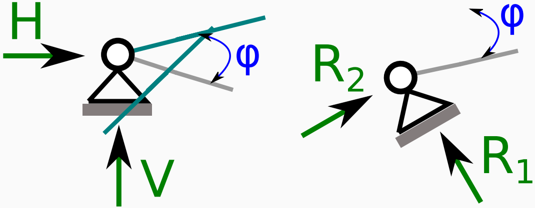
In the following…

… planar supports only are discussed (3 DOF for each rigid body)

- Roller: one reaction, translation and rotation allowed ($v = 1$)

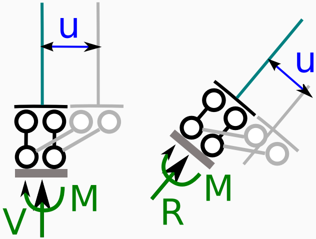


- Hinge (or pin): two reactions, rotation allowed ($v = 2$)

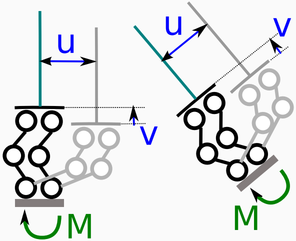


External supports (2/3)

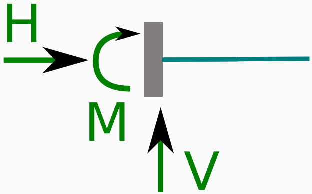
- Slider (or guide): two reactions, translation allowed (v = 2)

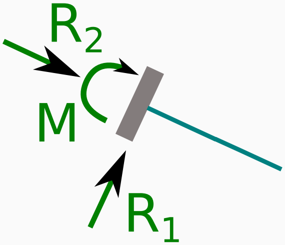


- Double slider: one reaction, translations allowed (v = 1)



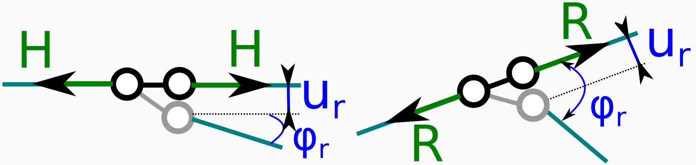
- Fixed end: three reactions, no translation or rotation allowed ($v = 3$)



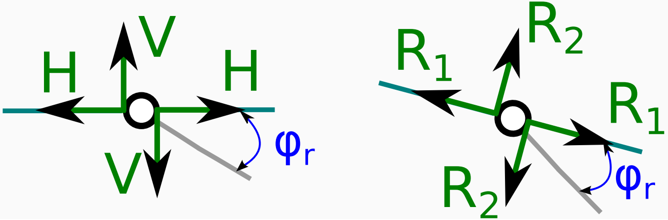


Connections between elements (1/2)

- Link: one pair of internal reactions, relative translation and rotation allowed ($v = 1$)

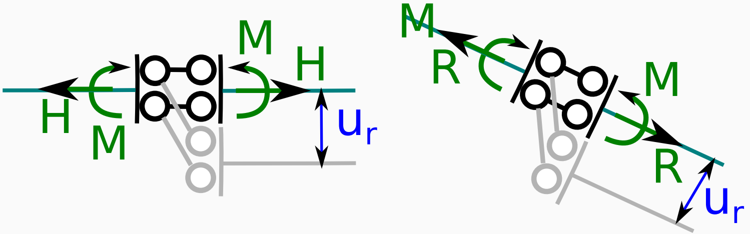


- Internal hinge: two pairs of reactions, relative rotation allowed ($v = 2$)

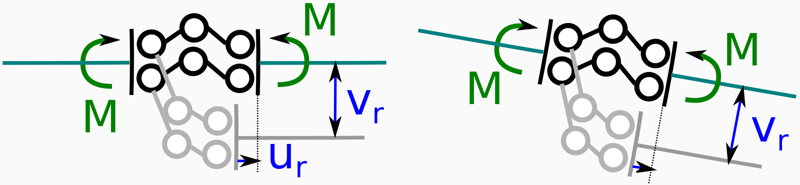


Connections between elements (2/2)

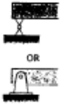
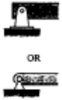
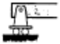
- Internal slider: two pairs of reactions, relative translation allowed ($v = 2$)



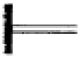
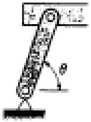
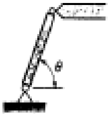
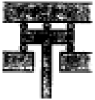
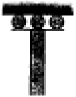
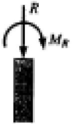
- Internal double slider: a pair of reactions, relative translations allowed ($v = 1$)



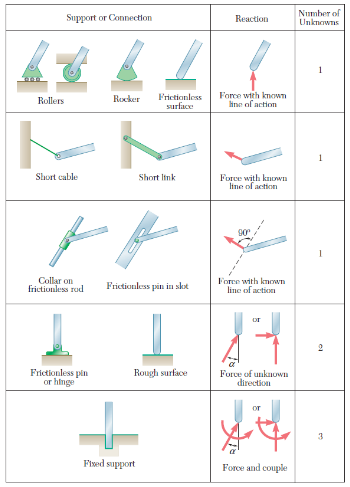
Other representations

Type	Sketch	Symbol	Movements Allowed or Prevented	Reaction Forces	Unknowns Created
(a) Pin			*Prevented:* horizontal translation, vertical translation *Allowed:* rotation	A single linear force of unknown direction or equivalently A horizontal force and a vertical force which are the components of the single force of unknown direction	
(b) Hinge			*Prevented:* relative displacement of member ends *Allowed:* both rotation and horizontal and vertical displacement	Equal and oppositely directed horizontal and vertical forces	
(c) Roller			*Prevented:* vertical translation *Allowed:* horizontal translation, rotation	A single linear force (either upward or downward*)	
(d) Rocker		OR			
(e) Elastomeric pad					

OTHER REPRESENTATIONS

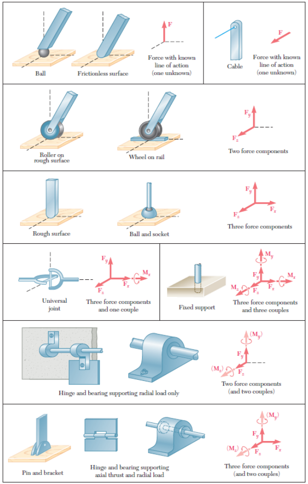
(*f*) Fixed end			*Prevented:* horizontal translation, vertical translation, rotation *Allowed:* none	Horizontal and vertical components of a linear resultant; moment	
(*g*) Link			*Prevented:* translation in the direction of link *Allowed:* translation perpendicular to link, rotation	A single linear force in the direction of the link	
(*h*) Guide			*Prevented:* vertical translation, rotation *Allowed:* horizontal translation	A single vertical linear force; moment	

Other 2D supports (Beer et al.)



Support or Connection	Reaction	Number of Unknowns
Rollers, Rocker, Frictionless surface	Force with known line of action	1
Short cable, Short link	Force with known line of action	1
Collar on frictionless rod, Frictionless pin in slot	90° Force with known line of action	1
Frictionless pin or hinge, Rough surface	α or Force of unknown direction	2
Fixed support	α or Force and couple	3

Three dimensional supports (Beer et al.)



- A structure is unstable if the total number of unknown reactive forces and moment components $r = \sum_i v_i$ is smaller than three times the number n of parts of structure members:

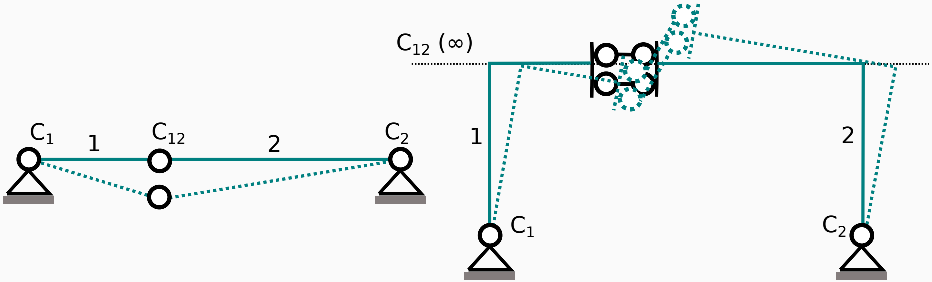
$$r < 3n$$

- A structure is stable if the total number of unknown reactive forces and moment components $r = \sum_i v_i$ is at least three times the number n of parts of structure members:

$$r \geq 3n$$

provided that member reactions are not concurrent or parallel or some of the components form a collapsible mechanism

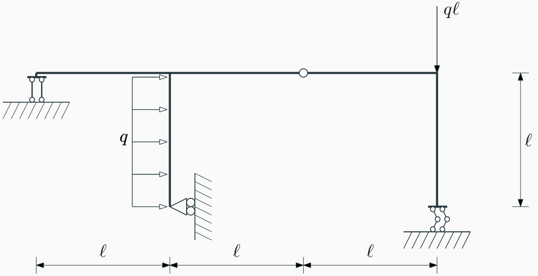
Unstable structures with $r = 3n$ (center C_1 for beam 1, C_2 for beam 2 and relative center C_{12} are aligned)



Unstable structures

Check carefully structures to find mechanism: unstable structures cannot, in general, withstand loads!

- A structure is statically determinate if the total number of unknown reactive forces and moment components r is equal to three times the number n of parts of structure members, i.e., $r = 3n$

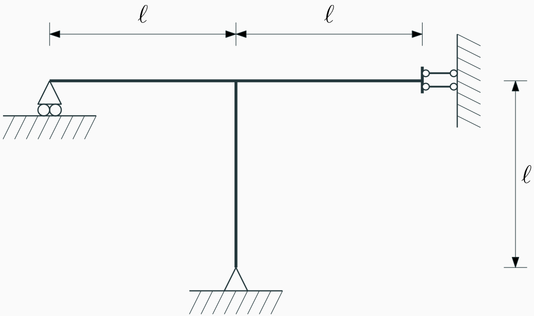


$$3n = 3 \times 2 = 6; \; r = \sum_i v_i = 2 + 1 + 2 + 1 = 6$$

Equilibrium is enough

The equilibrium equations are enough to solve the structure

- A structure is statically indeterminate if the total number of unknown reactive forces and moment components r is larger than three times the number n of parts of structure members, i.e., $r > 3n$



$$3n = 3 \times 1 = 3; r = \sum_i v_i = 2 + 1 + 2 = 5$$

Equilibrium is not enough

The equilibrium equations are not enough to solve the structure; compatibility conditions should be added

EQUILIBRIUM

Equations of static equilibrium

Equilibrium

A structure is considered to be in equilibrium if, initially at rest, remains at rest when subjected to a system of forces and couples

Whole structure and its parts

If a structure is in equilibrium, all its members and parts are also in equilibrium

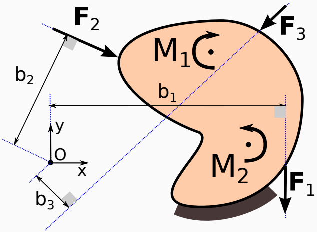
For a planar structure subjected to a coplanar system of N_F forces and N_M couples, the necessary and sufficient conditions for equilibrium are:

$$R_x = \sum_{j=1}^{N_F} F_{j,x} = F_{1,x} + F_{2,x} + \ldots = 0$$

$$R_y = \sum_{j=1}^{N_F} F_{j,y} = F_{1,y} + F_{2,y} + \ldots = 0$$

$$M_z = \sum_{k=1}^{N_F+N_M} M_{k,z} = \sum_{j=1}^{N_F} F_j b_j + \sum_{j=1}^{N_M} M_{j,z} =$$

$$= F_1\, b_1 + F_2\, b_2 + \ldots + M_1 + M_2 + \ldots = 0$$



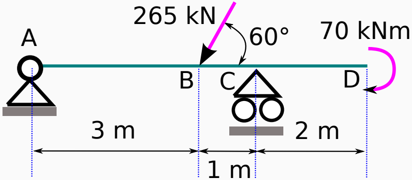
The meaning is that…

…the sum of all forces along x and y directions must be equal to zero and that the sum of all the moments (forces and couples) with respect to a certain point, must be zero too

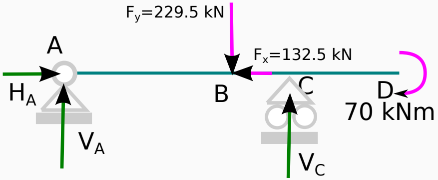
It is worth noting that:

- a moment is a force causing rotation without translation, defined as force F times lever arm b, i.e., $M = F \times b$
- a couple are two equal and opposite forces, F, separated by a moment arm, d (that is, two such forces that are not collinear), and therefore causing a moment, i.e., $M = F \times d$
- the arm: distance measured perpendicularly from each force to the chosen point

The objective is the calculation of the support reactions for the structure:



Structure



Free body diagram

The components of the inclined force are:

- $F_x = 265\,\text{kN} \times \cos 60° = 132.5\,\text{kN}$
- $F_y = 265\,\text{kN} \times \sin 60° = 229.5\,\text{kN}$

Considering the free body diagram (on the right):

- Equilibrium along horizontal direction:
  $H_A - F_x = 0$, i.e., $H_A = F_x = 132.5\,\text{kN}$,
- Equilibrium along vertical direction:
  $V_A + V_C - F_y = 0$, i.e., $V_A = F_y - V_C = 229.5\,\text{kN} - V_C$
- Equilibrium about point A:
  $V_A \times 0 + H_A \times 0 + V_C \times 4\,\text{m} - F_x \times 0 - F_y \times 3\,\text{m} - 70\,\text{kNm} = 0$, i.e.,
  $V_C = 189.6\,\text{kN}$

From the second equation (equilibrium along vertical direction), it is found
$V_A = 229.5\,\text{kN} - 189.6\,\text{kN} = 39.9\,\text{kN}$